Technical Appendix 1: The Optimization Problem

(Note on notation: Equation number (x) in the paper is referred to in the appendices as equation $(P.x)$.)

The optimization problem (see $(P.12) - (P.15)$ in the paper) is written as:

$$Max\ E_t[U(b_{t-1} + b_t) + \delta U(b_t + b_{t+1})] \tag{1}$$

$$subject\ to \tag{2}$$

$$c(e_t) + b_t \leq I_t(e_{t-1}, \alpha_{t-1}, b_{t-1} + b_{t-2}) - C \tag{3}$$

$$E_t(b_{t+1}) = E_t[I_{t+1}(e_t, \alpha_t, b_t + b_{t-1}) - c(e_{t+1})] - C \tag{4}$$

$$e_t \geq 0, b_t \geq 0 \tag{5}$$

where

$$U(B_t) = C + B_t \tag{6}$$

$$I_t(e_{t-1}, \alpha_{t-1}, B_{t-1}) = w_{1t}(e_{t-1}, \alpha_{t-1}) + w_{2t}(B_{t-1}) \tag{7}$$

$$B_t = b_{t-1} + b_t \tag{8}$$

Let EU_0 be the expected utility of households if they choose $e_t = e_L$ (with cost $c(e_L) = 0$) in t, given other parameters. Denote the corresponding spending on durables as b. Since income must equal expenditure, we have $I_t = C + b$. Then:

$$EU_0 = (C + b_{t-1} + b) + E_t[\delta(C + b + b_{t+1})] \tag{9}$$

or,

$$EU_0 = C(1 + \delta) + b_{t-1} + b(1 + \delta) + \delta E_t(b_{t+1}) \tag{10}$$

Note that:

$$b_{t+1} = I_{t+1}(e_t, \alpha_t, b_t + b_{t-1}) - c(e_{t+1}) - C \tag{11}$$

and

$$I_{t+1}(e_t, \alpha_t, B_t) = w_{1t+1}(e_t, \alpha_t) + w_{2t+1}(B_t) \tag{12}$$

Denote $w_{1t}(e_t, \alpha_t) + w_{2t}(B_t) = w(e_t, B_t, \alpha_t)$ in the exposition below.

Therefore, (from (10) and (11)) when households choose $e_t = e_L$:

$$E_t(b_{t+1}) = E_t[w_{t+1}(e_L, b + b_{t-1}, \alpha_t) - c(e_{t+1}) - C] \tag{13}$$

Combining (9) and (13) yields:

$$EU_0 = C + b(1+\delta) + b_{t-1} + \delta E_t[w_{t+1}(e_L, b + b_{t-1}, \alpha_t)] - \delta E_t[(c(e_{t+1})/e_t = e_L)] \tag{14}$$

Now, let EU_1 be the expected utility of households if they choose $e_t = e_H$ (with cost $c(e_H) = E$) in t, given other parameters. Then the corresponding spending on durables is $(b - E)$. Since income must equal expenditure, we have $I_t = C + E + (b - E)$. Then, from (1):

$$EU_1 = (C + b_{t-1} + b - E) + E_t[\delta(C + b - E + b_{t+1})] \tag{15}$$

Using arguments similar to those used to derive (10) – (14), we now get, assuming $e_t = e_H$ and $c(e_H) = E$:

$$EU_1 = C + b(1+\delta) + b_{t-1} - E(1+\delta) + \delta E_t[w_{t+1}(e_H, b - E + b_{t+1}, \alpha_t)] - \delta E_t[(e_{t+1}/e_t = e_H)] \tag{16}$$

From (14) and (16), we can derive:

$$EU_0 - EU_1 = E(1+\delta) + \delta[E_t w_{t+1}(e_L, b + b_{t-1}, \alpha_t) - E_t w_{t+1}(e_H, b - E + b_{t+1}, \alpha_t)] \tag{17}$$

$$-\delta[E_t\{c(e_{t+1})/e_t = e_L)\} - E_t\{c(e_{t+1}/e_t = e_H)\}] \tag{18}$$

Finally, note that:

$$E_t w_{t+1}(e_t, b + b_{t-1}, \alpha_t) = q_L[p_1 w_H + (1-p_1)w_L] + (1-q_L)[p_2 w_H + (1-p_2)w_L] + [\widetilde{p} w_H + (1-\widetilde{p})w_L] \tag{19}$$

and

$$E_t w_{t+1}(e_t, b - E + b_{t-1}, \alpha_t) = q_L[p_3 w_H + (1-p_3)w_L] + (1-q_L)[p_4 w_H + (1-p_4)w_L] + [\widetilde{p}_e w_H + (1-\widetilde{p}_e)w_L] \tag{20}$$

where

$$\widetilde{p} = \Phi_S(b_{t-1} + I_t - C) \tag{21}$$

$$\widetilde{p}_e = \Phi_S(b_{t-1} + I_t - C - E) \tag{22}$$

Also note that:

$$E_t[c(e_{t+1}/e_t) = E.\Pr(e_{t+1} = e_H/e_t) + 0.\Pr(e_{t+1} = e_L/e_t) \tag{23}$$

$$= E.\Pr(e_{t+1} = e_H/e_t) \tag{24}$$

Hence, using (19) – (22), we obtain:

$$E_t w_{t+1}(e_L, b + b_{t-1}, \alpha_t) - E_t w_{t+1}(e_H, b - E + b_{t+1}, \alpha_t) \tag{25}$$
$$= (w_H - w_L)[q_L(p_1 - p_3) + (1 - q_L)(p_2 - p_4) + (\widetilde{p} - \widetilde{p}_e)] \tag{26}$$

A household with income I_t will choose $e_t = e_H$ if and only if $EU_0 < EU_1$. Using (17) − (25), it is easy to see that the condition $EU_0 - EU_1 < 0$ reduces to equation $(P.16)$ in the paper:

$$(1 + \delta)E + \delta(w_H - w_L)[q_L(p_1 - p_3) + (1 - q_L)(p_2 - p_4) + (\widetilde{p} - \widetilde{p}_e)]... \tag{27}$$
$$-\delta E[\Pr(e_{t+1} = e_H/e_t = e_L) - \Pr(e_{t+1} = e_L/e_t = e_H)] < 0 \tag{28}$$

Technical Appendix 2: elements of (25 x 25) transition matrix P

Recall:

$$\beta_{m0} = (w_H + w_L) - c \tag{1}$$

$$\beta_{me} = (w_H + w_L) - c - e \tag{2}$$

$$\beta_{h0} = (2w_H) - c \tag{3}$$

$$\beta_{he} = (2w_H) - c - e \tag{4}$$

Define $\eta(I_t, b_{t-1}) = 1$ if a household with $t-$period income I_t and inherited durables b_{t-1} finds it optimal to choose $e_t = e_L$, and $\eta(I_t, b_{t-1})$ =0 otherwise (as per equation (P.16)).

Then define $t_1 - t_{10}$ as follows (where L, M, H denote income levels $2w_L$, $(w_L + w_H)$ and $2w_H$, respectively):

$$t_1 = 1 \ if \ \eta(M, 0) = 1 \tag{5}$$
$$= 0, \ otherwise \tag{6}$$

:

$$t_2 = 1 \ if \ \eta(M, \beta_{me}) = 1 \tag{7}$$
$$= 0, \ otherwise \tag{8}$$

$$t_3 = 1 \ if \ \eta(H, 0) = 1 \tag{9}$$
$$= 0, \ otherwise \tag{10}$$

$$t_4 = 1 \ if \ \eta(H, \beta_{me}) = 1 \tag{11}$$
$$= 0, \ otherwise \tag{12}$$

$$t_5 = 1 \ if \ \eta(H, \beta_{m0}) = 1 \tag{13}$$
$$= 0, \ otherwise \tag{14}$$

$$t_6 = 1 \ if \ \eta(H, \beta_{he}) = 1 \tag{15}$$
$$= 0, \ otherwise \tag{16}$$

$$t_7 = 1\ if\ \eta(H, \beta_{h0}) = 1 \tag{17}$$
$$= 0,\ otherwise \tag{18}$$

$$t_8 = 1\ if\ \eta(M, \beta_{m0}) = 1 \tag{19}$$
$$= 0,\ otherwise \tag{20}$$

$$t_9 = 1\ if\ \eta(M, \beta_{h0}) = 1 \tag{21}$$
$$= 0,\ otherwise \tag{22}$$

$$t_{10} = 1\ if\ \eta(M, \beta_{he}) = 1 \tag{23}$$
$$\tag{24}$$
$$= 0,\ otherwise \tag{25}$$

Using (1) – (24), we now define the elements of the transition matrix P.

Let $p_{i,j}$ be the $(i,j)^{th}$ element of the transition matrix P.
Column 1

$$p_{1,1} = q_L[\{1-p_1\}\{1-\Phi(0)\}] + (1-q_L)[\{1-p_2\}\{1-\Phi(0)\}] \tag{26}$$
$$p_{2,1} = q_L[\{1-p_1\}\{1-\Phi(\beta_{m0})\}] + (1-q_L)[\{1-p_2\}\{1-\Phi(\beta_{m0})\}] \tag{27}$$
$$p_{3,1} = q_L[\{1-p_1\}\{1-\Phi(\beta_{me})\}] + (1-q_L)[\{1-p_2\}\{1-\Phi(\beta_{me})\}] \tag{28}$$
$$p_{4,1} = q_L[\{1-p_1\}\{1-\Phi(\beta_{h0})\}] + (1-q_L)[\{1-p_2\}\{1-\Phi(\beta_{h0})\}] \tag{29}$$
$$p_{5,1} = q_L[\{1-p_1\}\{1-\Phi(\beta_{he})\}] + (1-q_L)[\{1-p_2\}\{1-\Phi(\beta_{he})\}] \tag{30}$$
$$p_{i,j} = 0\ \forall\ other\ i, j = 1 \tag{31}$$

Column 2

$$p_{6,2} = q_L[\{1-p_1\}\{1-\Phi(\beta_{m0})\}] + (1-q_L)[\{1-p_2\}\{1-\Phi(\beta_{m0})\}] \tag{32}$$
$$p_{7,2} = q_L[\{1-p_1\}\{1-\Phi(2\beta_{m0})\}] + (1-q_L)[\{1-p_2\}\{1-\Phi(2\beta_{m0})\}] \tag{33}$$
$$p_{8,2} = q_L[\{1-p_1\}\{1-\Phi(\beta_{m0}+\beta_{me})\}]+(1-q_L)[\{1-p_2\}\{1-\Phi(\beta_{m0}+\beta_{me})\}] \tag{34}$$
$$p_{9,2} = q_L[\{1-p_1\}\{1-\Phi(\beta_{m0}+\beta_{h0})\}]+(1-q_L)[\{1-p_2\}\{1-\Phi(\beta_{m0}+\beta_{h0})\}] \tag{35}$$
$$p_{10,2} = q_L[\{1-p_1\}\{1-\Phi(\beta_{m0}+\beta_{he})\}]+(1-q_L)[\{1-p_2\}\{1-\Phi(\beta_{m0}+\beta_{he})\}] \tag{36}$$
$$p_{i,j} = 0\ \forall\ other\ i, j = 2 \tag{37}$$

Column 3

$$p_{11,3} = q_L[\{1-p_3\}\{1-\Phi(\beta_{me})\}] + (1-q_L)[\{1-p_4\}\{1-\Phi(\beta_{me})\}] \tag{38}$$

$$p_{12,3} = q_L[\{1-p_3\}\{1-\Phi(\beta_{me}+\beta_{m0})\}] + (1-q_L)[\{1-p_4\}\{1-\Phi(\beta_{me}+\beta_{m0})\}] \tag{39}$$

$$p_{13,3} = q_L[\{1-p_3\}\{1-\Phi(2\beta_{me})\}] + (1-q_L)[\{1-p_4\}\{1-\Phi(2\beta_{me})\}] \tag{40}$$

$$p_{14,3} = q_L[\{1-p_3\}\{1-\Phi(\beta_{me}+\beta_{h0})\}] + (1-q_L)[\{1-p_4\}\{1-\Phi(\beta_{me}+\beta_{h0})\}] \tag{41}$$

$$p_{15,3} = q_L[\{1-p_3\}\{1-\Phi(\beta_{me}+\beta_{he})\}] + (1-q_L)[\{1-p_4\}\{1-\Phi(\beta_{me}+\beta_{he})\}] \tag{42}$$

$$p_{i,j} = 0 \ \forall \ other \ i, j = 3 \tag{43}$$

Column 4

$$p_{16,4} = q_L[\{1-p_1\}\{1-\Phi(\beta_{h0})\}] + (1-q_L)[\{1-p_2\}\{1-\Phi(\beta_{h0})\}] \tag{44}$$

$$p_{17,4} = q_L[\{1-p_1\}\{1-\Phi(\beta_{h0}+\beta_{m0})\}] + (1-q_L)[\{1-p_2\}\{1-\Phi(\beta_{h0}+\beta_{m0})\}] \tag{45}$$

$$p_{18,4} = q_L[\{1-p_1\}\{1-\Phi(\beta_{h0}+\beta_{me})\}] + (1-q_L)[\{1-p_2\}\{1-\Phi(\beta_{h0}+\beta_{me})\}] \tag{46}$$

$$p_{19,4} = q_L[\{1-p_1\}\{1-\Phi(2\beta_{h0})\}] + (1-q_L)[\{1-p_2\}\{1-\Phi(2\beta_{h0})\}] \tag{47}$$

$$p_{20,4} = q_L[\{1-p_1\}\{1-\Phi(\beta_{h0}+\beta_{he})\}] + (1-q_L)[\{1-p_2\}\{1-\Phi(\beta_{h0}+\beta_{he})\}] \tag{48}$$

$$p_{i,j} = 0 \ \forall \ other \ i, j = 4 \tag{49}$$

Column 5

$$p_{21,5} = q_L[\{1-p_3\}\{1-\Phi(\beta_{he})\}] + (1-q_L)[\{1-p_4\}\{1-\Phi(\beta_{he})\}] \tag{50}$$

$$p_{22,5} = q_L[\{1-p_3\}\{1-\Phi(\beta_{he}+\beta_{m0})\}] + (1-q_L)[\{1-p_4\}\{1-\Phi(\beta_{he}+\beta_{m0})\}] \tag{51}$$

$$p_{23,5} = q_L[\{1-p_3\}\{1-\Phi(\beta_{he}+\beta_{me})\}] + (1-q_L)[\{1-p_4\}\{1-\Phi(\beta_{he}+\beta_{me})\}] \tag{52}$$

$$p_{24,5} = q_L[\{1-p_3\}\{1-\Phi(\beta_{he}+\beta_{h0})\}] + (1-q_L)[\{1-p_4\}\{1-\Phi(\beta_{he}+\beta_{h0})\}] \tag{53}$$

$$p_{25,5} = q_L[\{1-p_3\}\{1-\Phi(2\beta_{he})\}] + (1-q_L)[\{1-p_4\}\{1-\Phi(2\beta_{he})\}] \tag{54}$$

Column 6

$$p_{1,6} = t_1[q_L\{(1-p_1)\Phi(0)+p_1((1-\Phi(0))\}+(1-q_L)\{(1-p_2)\Phi(0)+p_2(1-\Phi(0))\}] \quad (1)$$
$$p_{2,6} = t_1[q_L\{(1-p_1)\Phi(\beta_{m0})+p_1((1-\Phi(\beta_{m0}))\}+(1-q_L)\{(1-p_2)\Phi(\beta_{m0})+p_2(1-\Phi(\beta_{m0}))\}] \quad (2)$$
$$p_{3,6} = t_1[q_L\{(1-p_1)\Phi(\beta_{me})+p_1((1-\Phi(\beta_{me}))\}+(1-q_L)\{(1-p_2)\Phi(\beta_{me})+p_2(1-\Phi(\beta_{me}))\}] \quad (3)$$
$$p_{4,6} = t_1[q_L\{(1-p_1)\Phi(\beta_{h0})+p_1((1-\Phi(\beta_{h0}))\}+(1-q_L)\{(1-p_2)\Phi(\beta_{h0})+p_2(1-\Phi(\beta_{h0}))\}] \quad (4)$$
$$p_{5,6} = t_1[q_L\{(1-p_1)\Phi(\beta_{he})+p_1((1-\Phi(\beta_{he}))\}+(1-q_L)\{(1-p_2)\Phi(\beta_{he})+p_2(1-\Phi(\beta_{he}))\}] \quad (5)$$
$$p_{i,6} = 0 \ \forall \ other \ i \quad (6)$$

Column 7

$$p_{6,7} = t_8[q_L\{(1-p_1)\Phi(\beta_{m0})+p_1((1-\Phi(\beta_{m0}))\}+(1-q_L)\{(1-p_2)\Phi(\beta_{m0})+p_2(1-\Phi(\beta_{m0}))\}] \quad (7)$$
$$p_{7,7} = t_8[q_L\{(1-p_1)\Phi(2\beta_{m0})+p_1((1-\Phi(2\beta_{m0}))\}+(1-q_L)\{(1-p_2)\Phi(2\beta_{m0})+p_2(1-\Phi(2\beta_{m0}))\}] \quad (8)$$
$$p_{8,7} = t_8[q_L\{(1-p_1)\Phi(\beta_{m0}+\beta_{me})+p_1((1-\Phi(\beta_{m0}+\beta_{me}))\}+(1-q_L)\{(1-p_2)\Phi(\beta_{m0}+\beta_{me})+p_2(1-\Phi(\beta_{m0}+\beta_{me}))\}] \quad (9)$$
$$p_{9,7} = t_8[q_L\{(1-p_1)\Phi(\beta_{m0}+\beta_{h0})+p_1((1-\Phi(\beta_{m0}+\beta_{h0}))\}+(1-q_L)\{(1-p_2)\Phi(\beta_{m0}+\beta_{h0})+p_2(1-\Phi(\beta_{m0}+\beta_{h0}))\}] \quad (10)$$
$$p_{10,7} = t_8[q_L\{(1-p_1)\Phi(\beta_{m0}+\beta_{he})+p_1((1-\Phi(\beta_{m0}+\beta_{he}))\}+(1-q_L)\{(1-p_2)\Phi(\beta_{m0}+\beta_{he})+p_2(1-\Phi(\beta_{m0}+\beta_{he}))\}] \quad (11)$$
$$p_{i,7} = 0 \ \forall \ other \ i \quad (12)$$

Column 8

$$p_{11,8} = t_2[q_L\{(1-p_3)\Phi(\beta_{me})+p_3((1-\Phi(\beta_{me}))\}+(1-q_L)\{(1-p_4)\Phi(\beta_{me})+p_4(1-\Phi(\beta_{me}))\}] \quad (13)$$
$$p_{12,8} = t_2[q_L\{(1-p_3)\Phi(\beta_{me}+\beta_{m0})+p_3((1-\Phi(\beta_{me}+\beta_{m0}))\}+(1-q_L)\{(1-p_4)\Phi(\beta_{me}+\beta_{m0})+p_4(1-\Phi(\beta_{me}+\beta_{m0}))\} \quad (14)$$
$$p_{13,8} = t_2[q_L\{(1-p_3)\Phi(2\beta_{me})+p_3((1-\Phi(2\beta_{me}))\}+(1-q_L)\{(1-p_4)\Phi(2\beta_{me})+p_4(1-\Phi(2\beta_{me}))\}] \quad (15)$$
$$p_{14,8} = t_2[q_L\{(1-p_3)\Phi(\beta_{me}+\beta_{h0})+p_3((1-\Phi(\beta_{me}+\beta_{h0}))\}+(1-q_L)\{(1-p_4)\Phi(\beta_{me}+\beta_{h0})+p_4(1-\Phi(\beta_{me}+\beta_{h0}))\}] \quad (16)$$
$$p_{15,8} = t_2[q_L\{(1-p_3)\Phi(\beta_{me}+\beta_{he})+p_3((1-\Phi(\beta_{me}+\beta_{he}))\}+(1-q_L)\{(1-p_4)\Phi(\beta_{me}+\beta_{he})+p_4(1-\Phi(\beta_{me}+\beta_{he}))\}] \quad (17)$$
$$p_{i,8} = 0 \ \forall \ other \ i \quad (18)$$

Column 9

$$p_{16,9} = t_9[q_L\{(1{-}p_1)\Phi(\beta_{h0}){+}p_1((1{-}\Phi(\beta_{h0}))\}{+}(1{-}q_L)\{(1{-}p_2)\Phi(\beta_{h0}){+}p_2(1{-}\Phi(\beta_{h0}))\}] \tag{19}$$

$$p_{17,9} = t_9[q_L\{(1{-}p_1)\Phi(\beta_{h0}{+}\beta_{m0}){+}p_1((1{-}\Phi(\beta_{h0}{+}\beta_{m0}))\}{+}(1{-}q_L)\{(1{-}p_2)\Phi(\beta_{h0}{+}\beta_{m0}){+}p_2(1{-}\Phi(\beta_{h0}{+}\beta_{m0}))\}] \tag{20}$$

$$p_{18,9} = t_9[q_L\{(1{-}p_1)\Phi(\beta_{h0}{+}\beta_{me}){+}p_1((1{-}\Phi(\beta_{h0}{+}\beta_{me}))\}{+}(1{-}q_L)\{(1{-}p_2)\Phi(\beta_{h0}{+}\beta_{me}){+}p_2(1{-}\Phi(\beta_{h0}{+}\beta_{me}))\}] \tag{21}$$

$$p_{19,9} = t_9[q_L\{(1{-}p_1)\Phi(2\beta_{h0}){+}p_1((1{-}\Phi(2\beta_{h0}))\}{+}(1{-}q_L)\{(1{-}p_2)\Phi(2\beta_{h0}){+}p_2(1{-}\Phi(2\beta_{h0}))\}] \tag{22}$$

$$p_{20,9} = t_9[q_L\{(1{-}p_1)\Phi(\beta_{h0}{+}\beta_{he}){+}p_1((1{-}\Phi(\beta_{h0}{+}\beta_{he}))\}{+}(1{-}q_L)\{(1{-}p_2)\Phi(\beta_{h0}{+}\beta_{he}){+}p_2(1{-}\Phi(\beta_{h0}{+}\beta_{he}))\}] \tag{23}$$

$$p_{i,9} = 0 \ \forall \ other \ i \tag{24}$$

Column 10

$$p_{21,10} = t_{10}[q_L\{(1{-}p_3)\Phi(\beta_{he}){+}p_3((1{-}\Phi(\beta_{he}))\}{+}(1{-}q_L)\{(1{-}p_4)\Phi(\beta_{he}){+}p_4(1{-}\Phi(\beta_{he}))\}] \tag{25}$$

$$p_{22,10} = t_{10}[q_L\{(1{-}p_3)\Phi(\beta_{he}{+}\beta_{m0}){+}p_3((1{-}\Phi(\beta_{he}{+}\beta_{m0}))\}{+}(1{-}q_L)\{(1{-}p_4)\Phi(\beta_{he}{+}\beta_{m0}){+}p_4(1{-}\Phi(\beta_{he}{+}\beta_{m0}))\} \tag{26}$$

$$p_{23,10} = t_{10}[q_L\{(1{-}p_3)\Phi(\beta_{he}{+}\beta_{me}){+}p_3((1{-}\Phi(\beta_{he}{+}\beta_{me}))\}{+}(1{-}q_L)\{(1{-}p_4)\Phi(\beta_{he}{+}\beta_{me}){+}p_4(1{-}\Phi(\beta_{he}{+}\beta_{me}))\} \tag{27}$$

$$p_{24,10} = t_{10}[q_L\{(1{-}p_3)\Phi(\beta_{he}{+}\beta_{h0}){+}p_3((1{-}\Phi(\beta_{he}{+}\beta_{h0}))\}{+}(1{-}q_L)\{(1{-}p_4)\Phi(\beta_{he}{+}\beta_{h0}){+}p_4(1{-}\Phi(\beta_{he}{+}\beta_{h0}))\}] \tag{28}$$

$$p_{25,10} = t_{10}[q_L\{(1{-}p_3)\Phi(2\beta_{he}){+}p_3((1{-}\Phi(2\beta_{he}))\}{+}(1{-}q_L)\{(1{-}p_4)\Phi(2\beta_{he}){+}p_4(1{-}\Phi(2\beta_{he}))\}] \tag{29}$$

$$p_{i,10} = 0 \ \forall \ other \ i \tag{30}$$

Column 11: **Let** $\iota = (1 - \mathbf{t}_1)$ **in column 11**

$$p_{1,11} = \iota[q_L\{(1-p_1)\Phi(0)+p_1((1-\Phi(0))\}+(1-q_L)\{(1-p_2)\Phi(0)+p_2(1-\Phi(0))\}] \tag{31}$$
$$p_{2,11} = \iota[q_L\{(1-p_1)\Phi(\beta_{m0})+p_1((1-\Phi(\beta_{m0}))\}+(1-q_L)\{(1-p_2)\Phi(\beta_{m0})+p_2(1-\Phi(\beta_{m0}))\}] \tag{32}$$
$$p_{3,11} = \iota[q_L\{(1-p_1)\Phi(\beta_{me})+p_1((1-\Phi(\beta_{me}))\}+(1-q_L)\{(1-p_2)\Phi(\beta_{me})+p_2(1-\Phi(\beta_{me}))\}] \tag{33}$$
$$p_{4,11} = \iota[q_L\{(1-p_1)\Phi(\beta_{h0})+p_1((1-\Phi(\beta_{h0}))\}+(1-q_L)\{(1-p_2)\Phi(\beta_{h0})+p_2(1-\Phi(\beta_{h0}))\}] \tag{34}$$
$$p_{5,11} = \iota[q_L\{(1-p_1)\Phi(\beta_{he})+p_1((1-\Phi(\beta_{he}))\}+(1-q_L)\{(1-p_2)\Phi(\beta_{he})+p_2(1-\Phi(\beta_{he}))\}] \tag{35}$$
$$p_{i,11} = 0 \; \forall \; other \; i \tag{36}$$

Column 12: **Let** $\iota = (1 - \mathbf{t}_8)$ **in column 12**

$$p_{6,12} = \iota[q_L\{(1-p_1)\Phi(\beta_{m0})+p_1((1-\Phi(\beta_{m0}))\}+(1-q_L)\{(1-p_2)\Phi(\beta_{m0})+p_2(1-\Phi(\beta_{m0}))\}] \tag{37}$$
$$p_{7,12} = \iota[q_L\{(1-p_1)\Phi(2\beta_{m0})+p_1((1-\Phi(2\beta_{m0}))\}+(1-q_L)\{(1-p_2)\Phi(2\beta_{m0})+p_2(1-\Phi(2\beta_{m0}))\}] \tag{38}$$
$$p_{8,12} = \iota[q_L\{(1-p_1)\Phi(\beta_{m0}+\beta_{me})+p_1((1-\Phi(\beta_{m0}+\beta_{me}))\}+(1-q_L)\{(1-p_2)\Phi(\beta_{m0}+\beta_{me})+p_2(1-\Phi(\beta_{m0}+\beta_{me}))\}] \tag{39}$$
$$p_{9,12} = \iota[q_L\{(1-p_1)\Phi(\beta_{m0}+\beta_{h0})+p_1((1-\Phi(\beta_{m0}+\beta_{h0}))\}+(1-q_L)\{(1-p_2)\Phi(\beta_{m0}+\beta_{h0})+p_2(1-\Phi(\beta_{m0}+\beta_{h0}))\}] \tag{40}$$
$$p_{10,12} = \iota[q_L\{(1-p_1)\Phi(\beta_{m0}+\beta_{he})+p_1((1-\Phi(\beta_{m0}+\beta_{he}))\}+(1-q_L)\{(1-p_2)\Phi(\beta_{m0}+\beta_{he})+p_2(1-\Phi(\beta_{m0}+\beta_{he}))\}] \tag{41}$$
$$p_{i,12} = 0 \; \forall \; other \; i \tag{42}$$

Column 13: **Let** $\iota = (1 - \mathbf{t}_2)$ **in column 13**

$$p_{11,13} = \iota[q_L\{(1-p_3)\Phi(\beta_{me})+p_3((1-\Phi(\beta_{me}))\}+(1-q_L)\{(1-p_4)\Phi(\beta_{me})+p_4(1-\Phi(\beta_{me}))\}] \tag{43}$$
$$p_{12,13} = \iota[q_L\{(1-p_3)\Phi(\beta_{me}+\beta_{m0})+p_3((1-\Phi(\beta_{me}+\beta_{m0}))\}+(1-q_L)\{(1-p_4)\Phi(\beta_{me}+\beta_{m0})+p_4(1-\Phi(\beta_{me}+\beta_{m0}))\} \tag{44}$$
$$p_{13,13} = \iota[q_L\{(1-p_3)\Phi(2\beta_{me})+p_3((1-\Phi(2\beta_{me}))\}+(1-q_L)\{(1-p_4)\Phi(2\beta_{me})+p_4(1-\Phi(2\beta_{me}))\}] \tag{45}$$
$$p_{14,13} = \iota[q_L\{(1-p_3)\Phi(\beta_{me}+\beta_{h0})+p_3((1-\Phi(\beta_{me}+\beta_{h0}))\}+(1-q_L)\{(1-p_4)\Phi(\beta_{me}+\beta_{h0})+p_4(1-\Phi(\beta_{me}+\beta_{h0}))\}] \tag{46}$$
$$p_{15,13} = \iota[q_L\{(1-p_3)\Phi(\beta_{me}+\beta_{he})+p_3((1-\Phi(\beta_{me}+\beta_{he}))\}+(1-q_L)\{(1-p_4)\Phi(\beta_{me}+\beta_{he})+p_4(1-\Phi(\beta_{me}+\beta_{he}))\}] \tag{47}$$

$$p_{i,13} = 0 \; \forall \; other \; i \tag{48}$$

Column 14: **Let $\iota = (1 - \mathbf{t}_9)$ in column 14**

$$p_{16,14} = \iota[q_L\{(1-p_1)\Phi(\beta_{h0})+p_1((1-\Phi(\beta_{h0}))\}+(1-q_L)\{(1-p_2)\Phi(\beta_{h0})+p_2(1-\Phi(\beta_{h0}))\}] \tag{49}$$
$$p_{17,14} = \iota[q_L\{(1-p_1)\Phi(\beta_{h0}+\beta_{m0})+p_1((1-\Phi(\beta_{h0}+\beta_{m0}))\}+(1-q_L)\{(1-p_2)\Phi(\beta_{h0}+\beta_{m0})+p_2(1-\Phi(\beta_{h0}+\beta_{m0}))\}] \tag{50}$$
$$p_{18,14} = \iota[q_L\{(1-p_1)\Phi(\beta_{h0}+\beta_{me})+p_1((1-\Phi(\beta_{h0}+\beta_{me}))\}+(1-q_L)\{(1-p_2)\Phi(\beta_{h0}+\beta_{me})+p_2(1-\Phi(\beta_{h0}+\beta_{me}))\}] \tag{51}$$
$$p_{19,14} = \iota[q_L\{(1-p_1)\Phi(2\beta_{h0})+p_1((1-\Phi(2\beta_{h0}))\}+(1-q_L)\{(1-p_2)\Phi(2\beta_{h0})+p_2(1-\Phi(2\beta_{h0}))\}] \tag{52}$$
$$p_{20,14} = \iota[q_L\{(1-p_1)\Phi(\beta_{h0}+\beta_{he})+p_1((1-\Phi(\beta_{h0}+\beta_{he}))\}+(1-q_L)\{(1-p_2)\Phi(\beta_{h0}+\beta_{he})+p_2(1-\Phi(\beta_{h0}+\beta_{he}))\}] \tag{53}$$
$$p_{i,14} = 0 \; \forall \; other \; i \tag{54}$$

Column 15: **Let $\iota = (1 - \mathbf{t}_{10})$ in column 15**

$$p_{21,15} = \iota[q_L\{(1-p_3)\Phi(\beta_{he})+p_3((1-\Phi(\beta_{he}))\}+(1-q_L)\{(1-p_4)\Phi(\beta_{he})+p_4(1-\Phi(\beta_{he}))\}] \tag{55}$$
$$p_{22,15} = \iota[q_L\{(1-p_3)\Phi(\beta_{he}+\beta_{m0})+p_3((1-\Phi(\beta_{he}+\beta_{m0}))\}+(1-q_L)\{(1-p_4)\Phi(\beta_{he}+\beta_{m0})+p_4(1-\Phi(\beta_{he}+\beta_{m0}))\}] \tag{56}$$
$$p_{23,15} = \iota[q_L\{(1-p_3)\Phi(\beta_{he}+\beta_{me})+p_3((1-\Phi(\beta_{he}+\beta_{me}))\}+(1-q_L)\{(1-p_4)\Phi(\beta_{he}+\beta_{me})+p_4(1-\Phi(\beta_{he}+\beta_{me}))\}] \tag{57}$$
$$p_{24,15} = \iota[q_L\{(1-p_3)\Phi(\beta_{he}+\beta_{h0})+p_3((1-\Phi(\beta_{he}+\beta_{h0}))\}+(1-q_L)\{(1-p_4)\Phi(\beta_{he}+\beta_{h0})+p_4(1-\Phi(\beta_{he}+\beta_{h0}))\}] \tag{58}$$
$$p_{25,15} = \iota[q_L\{(1-p_3)\Phi(2\beta_{he})+p_3((1-\Phi(2\beta_{he}))\}+(1-q_L)\{(1-p_4)\Phi(2\beta_{he})+p_4(1-\Phi(2\beta_{he}))\}] \tag{59}$$
$$p_{i,15} = 0 \; \forall \; other \; i \tag{60}$$

Column 16:

$$p_{1,16} = t_3[q_L p_1 \Phi(0) + (1 - q_L) p_2 \Phi(0)] \tag{1}$$
$$p_{2,16} = t_3[q_L p_1 \Phi(\beta_{m0}) + (1 - q_L) p_2 \Phi(\beta_{m0})] \tag{2}$$
$$p_{3,16} = t_3[q_L p_1 \Phi(\beta_{me}) + (1 - q_L) p_2 \Phi(\beta_{me})] \tag{3}$$
$$p_{4,16} = t_3[q_L p_1 \Phi(\beta_{h0}) + (1 - q_L) p_2 \Phi(\beta_{h0})] \tag{4}$$
$$p_{5,16} = t_3[q_L p_1 \Phi(\beta_{he}) + (1 - q_L) p_2 \Phi(\beta_{he})] \tag{5}$$
$$p_{i,16} = 0 \ \forall \ other \ i \tag{6}$$

Column 17:

$$p_{6,17} = t_5[q_L p_1 \Phi(\beta_{m0}) + (1 - q_L) p_2 \Phi(\beta_{m0})] \tag{7}$$
$$p_{7,17} = t_5[q_L p_1 \Phi(2\beta_{m0}) + (1 - q_L) p_2 \Phi(2\beta_{m0})] \tag{8}$$
$$p_{8,17} = t_5[q_L p_1 \Phi(\beta_{m0} + \beta_{me}) + (1 - q_L) p_2 \Phi(\beta_{m0} + \beta_{me})] \tag{9}$$
$$p_{9,17} = t_5[q_L p_1 \Phi(\beta_{m0} + \beta_{h0}) + (1 - q_L) p_2 \Phi(\beta_{m0} + \beta_{h0})] \tag{10}$$
$$p_{10,17} = t_5[q_L p_1 \Phi(\beta_{m0} + \beta_{he}) + (1 - q_L) p_2 \Phi(\beta_{m0} + \beta_{he})] \tag{11}$$
$$p_{i,17} = 0 \ \forall \ other \ i \tag{12}$$

Column 18:

$$p_{11,18} = t_4[q_L p_3 \Phi(\beta_{me}) + (1 - q_L) p_4 \Phi(\beta_{me})] \tag{13}$$
$$p_{12,18} = t_4[q_L p_3 \Phi(\beta_{me} + \beta_{m0}) + (1 - q_L) p_4 \Phi(\beta_{me} + \beta_{m0})] \tag{14}$$
$$p_{13,18} = t_4[q_L p_3 \Phi(2\beta_{me}) + (1 - q_L) p_4 \Phi(2\beta_{me})] \tag{15}$$
$$p_{14,18} = t_4[q_L p_3 \Phi(\beta_{me} + \beta_{h0}) + (1 - q_L) p_4 \Phi(\beta_{me} + \beta_{h0})] \tag{16}$$
$$p_{15,18} = t_4[q_L p_3 \Phi(\beta_{me} + \beta_{he}) + (1 - q_L) p_4 \Phi(\beta_{me} + \beta_{he})] \tag{17}$$
$$p_{i,18} = 0 \ \forall \ other \ i \tag{18}$$

Column 19:

$$p_{16,19} = t_7[q_L p_1 \Phi(\beta_{h0}) + (1 - q_L) p_2 \Phi(\beta_{h0})] \tag{19}$$
$$p_{17,19} = t_7[q_L p_1 \Phi(\beta_{h0} + \beta_{m0}) + (1 - q_L) p_2 \Phi(\beta_{h0} + \beta_{m0})] \tag{20}$$
$$p_{18,19} = t_7[q_L p_1 \Phi(\beta_{h0} + \beta_{me}) + (1 - q_L) p_2 \Phi(\beta_{h0} + \beta_{me})] \tag{21}$$
$$p_{19,19} = t_7[q_L p_1 \Phi(2\beta_{h0}) + (1 - q_L) p_2 \Phi(2\beta_{h0})] \tag{22}$$
$$p_{20,19} = t_7[q_L p_1 \Phi(\beta_{h0} + \beta_{he}) + (1 - q_L) p_2 \Phi(\beta_{h0} + \beta_{he})] \tag{23}$$
$$p_{i,19} = 0 \ \forall \ other \ i \tag{24}$$

Column 20:

$$p_{21,20} = t_6[q_L p_3 \Phi(\beta_{he}) + (1 - q_L) p_4 \Phi(\beta_{he})] \tag{25}$$
$$p_{22,20} = t_6[q_L p_3 \Phi(\beta_{he} + \beta_{m0}) + (1 - q_L) p_4 \Phi(\beta_{he} + \beta_{m0})] \tag{26}$$
$$p_{23,20} = t_6[q_L p_3 \Phi(\beta_{he} + \beta_{me}) + (1 - q_L) p_4 \Phi(\beta_{he} + \beta_{me})] \tag{27}$$
$$p_{24,20} = t_6[q_L p_3 \Phi(\beta_{he} + \beta_{h0}) + (1 - q_L) p_4 \Phi(\beta_{he} + \beta_{h0})] \tag{28}$$
$$p_{25,20} = t_6[q_L p_3 \Phi(2\beta_{he}) + (1 - q_L) p_4 \Phi(2\beta_{he})] \tag{29}$$
$$p_{i,20} = 0 \; \forall \; other \; i \tag{30}$$

Column 21: **Let** $\iota = (1 - \mathbf{t}_3)$ **in column 21**

$$p_{1,21} = \iota[q_L p_1 \Phi(0) + (1 - q_L) p_2 \Phi(0)] \tag{31}$$
$$p_{2,21} = \iota[q_L p_1 \Phi(\beta_{m0}) + (1 - q_L) p_2 \Phi(\beta_{m0})] \tag{32}$$
$$p_{3,21} = \iota[q_L p_1 \Phi(\beta_{me}) + (1 - q_L) p_2 \Phi(\beta_{me})] \tag{33}$$
$$p_{4,21} = \iota[q_L p_1 \Phi(\beta_{h0}) + (1 - q_L) p_2 \Phi(\beta_{h0})] \tag{34}$$
$$p_{5,21} = \iota[q_L p_1 \Phi(\beta_{he}) + (1 - q_L) p_2 \Phi(\beta_{he})] \tag{35}$$
$$p_{i,21} = 0 \; \forall \; other \; i \tag{36}$$

Column 22: **Let** $\iota = (1 - \mathbf{t}_5)$ **in column 22**

$$p_{6,22} = \iota[q_L p_1 \Phi(\beta_{m0}) + (1 - q_L) p_2 \Phi(\beta_{m0})] \tag{37}$$
$$p_{7,22} = \iota[q_L p_1 \Phi(2\beta_{m0}) + (1 - q_L) p_2 \Phi(2\beta_{m0})] \tag{38}$$
$$p_{8,22} = \iota[q_L p_1 \Phi(\beta_{m0} + \beta_{me}) + (1 - q_L) p_2 \Phi(\beta_{m0} + \beta_{me})] \tag{39}$$
$$p_{9,22} = \iota[q_L p_1 \Phi(\beta_{m0} + \beta_{h0}) + (1 - q_L) p_2 \Phi(\beta_{m0} + \beta_{h0})] \tag{40}$$
$$p_{10,22} = \iota[q_L p_1 \Phi(\beta_{m0} + \beta_{he}) + (1 - q_L) p_2 \Phi(\beta_{m0} + \beta_{he})] \tag{41}$$
$$p_{i,22} = 0 \; \forall \; other \; i \tag{42}$$

Column 23: **Let** $\iota = (1 - \mathbf{t}_4)$ **in column 23**

$$p_{11,23} = \iota[q_L p_3 \Phi(\beta_{me}) + (1 - q_L) p_4 \Phi(\beta_{me})] \tag{43}$$
$$p_{12,23} = \iota[q_L p_3 \Phi(\beta_{me} + \beta_{m0}) + (1 - q_L) p_4 \Phi(\beta_{me} + \beta_{m0})] \tag{44}$$
$$p_{13,23} = \iota[q_L p_3 \Phi(2\beta_{me}) + (1 - q_L) p_4 \Phi(2\beta_{me})] \tag{45}$$
$$p_{14,23} = \iota[q_L p_3 \Phi(\beta_{me} + \beta_{h0}) + (1 - q_L) p_4 \Phi(\beta_{me} + \beta_{h0})] \tag{46}$$
$$p_{15,23} = \iota[q_L p_3 \Phi(\beta_{me} + \beta_{he}) + (1 - q_L) p_4 \Phi(\beta_{me} + \beta_{he})] \tag{47}$$
$$p_{i,23} = 0 \; \forall \; other \; i \tag{48}$$

Column 24: **Let** $\iota = (1 - \mathbf{t}_7)$ **in column 24**

$$p_{16,24} = \iota[q_L p_1 \Phi(\beta_{h0}) + (1 - q_L) p_2 \Phi(\beta_{h0})] \tag{49}$$

$$p_{17,24} = \iota[q_L p_1 \Phi(\beta_{h0} + \beta_{m0}) + (1 - q_L) p_2 \Phi(\beta_{h0} + \beta_{m0})] \tag{50}$$

$$p_{18,24} = \iota[q_L p_1 \Phi(\beta_{h0} + \beta_{me}) + (1 - q_L) p_2 \Phi(\beta_{h0} + \beta_{me})] \tag{51}$$

$$p_{19,24} = \iota[q_L p_1 \Phi(2\beta_{h0}) + (1 - q_L) p_2 \Phi(2\beta_{h0})] \tag{52}$$

$$p_{20,24} = \iota[q_L p_1 \Phi(\beta_{h0} + \beta_{he}) + (1 - q_L) p_2 \Phi(\beta_{h0} + \beta_{he})] \tag{53}$$

$$p_{i,24} = 0 \; \forall \; other \;\; i \tag{54}$$

Column 25: **Let** $\iota = (1 - \mathbf{t}_6)$ **in column 25**

$$p_{21,25} = \iota[q_L p_3 \Phi(\beta_{he}) + (1 - q_L) p_4 \Phi(\beta_{he})] \tag{55}$$

$$p_{22,25} = \iota[q_L p_3 \Phi(\beta_{he} + \beta_{m0}) + (1 - q_L) p_4 \Phi(\beta_{he} + \beta_{m0})] \tag{56}$$

$$p_{23,25} = \iota[q_L p_3 \Phi(\beta_{he} + \beta_{me}) + (1 - q_L) p_4 \Phi(\beta_{he} + \beta_{me})] \tag{57}$$

$$p_{24,25} = \iota[q_L p_3 \Phi(\beta_{he} + \beta_{h0}) + (1 - q_L) p_4 \Phi(\beta_{he} + \beta_{h0})] \tag{58}$$

$$p_{25,25} = \iota[q_L p_3 \Phi(2\beta_{he}) + (1 - q_L) p_4 \Phi(2\beta_{he})] \tag{59}$$

$$p_{i,25} = 0 \; \forall \; other \;\; i \tag{60}$$

Technical Appendix 3: Unique values of total durables corresponding to 25 states

Recall that:

$$\beta_{m0} = w_H + w_L - c \quad (1)$$
$$\beta_{me} = w_H + w_L - c - e \quad (2)$$
$$\beta_{h0} = 2w_H - c \quad (3)$$
$$\beta_{he} = 2w_H - c - e \quad (4)$$

In the table below, we list the 25 different states that are possible in our model and the total household durables corresponding to each state (sum of elements of the state). Only unique values of total household durables are listed (recall that state (a, b) has the same total durables as state (b, a)).

Serial No.	State (β_{t-1}, β_t)	Unique values of household durables in time t
1	$(0, 0)$	0
2	$(0, \beta_{m0})$	β_{m0}
3	$(0, \beta_{me})$	β_{me}
4	$(0, \beta_{h0})$	β_{h0}
5	$(0, \beta_{he})$	β_{he}
6	$(\beta_{m0}, 0)$	$-$
7	(β_{m0}, β_{m0})	$2\beta_{m0}$
8	(β_{m0}, β_{me})	$\beta_{m0} + \beta_{me}$
9	(β_{m0}, β_{h0})	$\beta_{m0} + \beta_{h0}$
10	(β_{m0}, β_{he})	$\beta_{m0} + \beta_{he}$
11	$(\beta_{me}, 0)$	$-$
12	(β_{me}, β_{m0})	$-$
13	(β_{me}, β_{me})	$2\beta_{me}$
14	(β_{me}, β_{h0})	$\beta_{me} + \beta_{h0}$
15	(β_{me}, β_{he})	$\beta_{me} + \beta_{he}$
16	$(\beta_{h0}, 0)$	$-$
17	(β_{h0}, β_{m0})	$-$
18	(β_{h0}, β_{me})	$-$
19	(β_{h0}, β_{h0})	$2\beta_{h0}$
20	(β_{h0}, β_{he})	$\beta_{h0} + \beta_{he}$
21	$(\beta_{he}, 0)$	$-$
22	(β_{he}, β_{m0})	$-$
23	(β_{he}, β_{me})	$-$
24	(β_{he}, β_{h0})	$-$
25	(β_{he}, β_{he})	$2\beta_{he}$

The table above shows that there are 15 unique values of household durables that can possibly exist at any time.

Now, note that under the assumption $c = 2w_L$, the following conditions are also true:

$$\beta_{h0} = 2\beta_{m0} \tag{5}$$

$$\beta_{he} = \beta_{m0} + \beta_{me} \tag{6}$$

$$\beta_{m0} + \beta_{he} = \beta_{me} + \beta_{h0} \tag{7}$$

This reduces the number of unique values of total durables by 3. Hence there are 12 unique values of total durables that are possible in our model.

Technical Appendix 4: Example 1

This appendix outlines the algorithm used to solve for steady state equilibrium, given certain parameter values. The (STATA) code that does this is available from the author upon request.

(Note on notation: Equation number (x) in the paper is referred to in the appendices as equation $(P.x)$.)

Recall that in any period t, households may choose (depending on their income I_t and how much durables b_{t-1} they inherited) durables of value $0, \beta_{m0}, \beta_{me}, \beta_{h0}$ or β_{he} durables, where

$$\beta_{m0} = (w_H + w_L) - c \tag{1}$$

$$\beta_{me} = (w_H + w_L) - c - e \tag{2}$$

$$\beta_{h0} = (2w_H) - c \tag{3}$$

$$\beta_{he} = (2w_H) - c - e \tag{4}$$

The total durables in a household at the end of period t is the sum of durables inherited (b_{t-1}) and that chosen in period t, (b_t), where $b_{t-1} \in \{0, \beta_{m0}, \beta_{me}, \beta_{h0}, \beta_{he}\}$ and $b_t \in \{0, \beta_{m0}, \beta_{me}, \beta_{h0}, \beta_{he}\}$. This indicates that there are 25 distinct realizations of the state (b_{t-1}, b_t). In each period t, b_t is chosen based on income (L, M, H), inherited durables (b_{t-1}) and expectations about future education spending $(E_t c(e_{t+1}))$ as laid out in equation $(P.16)$, viz:

$$\begin{aligned}(1+\delta)E + \delta(w_H - w_L)[q_L(p_1 - p_3) + (1 - q_L)(p_2 - p_4) + (\widetilde{p} - \widetilde{p}_e)]... \\ - \{\Pr(e_{t+1} = E/e_t = e_L) - \Pr(e_{t+1} = E/e_t = e_H)\}\delta E < 0\end{aligned} \tag{5}$$

where $\widetilde{p} = \Phi_S(b_{t-1} + I_t - C)$ and $\widetilde{p}_e = \Phi_S(b_{t-1} + I_t - C - E)$ are the values of the signal function under $S = (\beta, \sigma^2)$ when e_L (with cost 0) or e_H (with cost E) is chosen, respectively.

Recall that in our model, decisions (b_t) are made based on $(P.16)$ (given b_{t-1}) and future incomes I_{t+1} depend on these decisions, which in turn determine future decisions (b_{t+1}). This defines a stochastic process:

$$x_t P = x_{t+1} \tag{6}$$

where x_t denotes the number of households in state (b_t, b_{t-1}), and P is the transition matrix denoting probabilities of moving from any state (b_t, b_{t-1}) to (b_{t+1}, b_t).

Our goal is to derive the transition matrix P (based on parameter values and $(P.16)$) and obtain the steady state distribution of households $x*$. We describe our algorithm for doing this below.

For simplicity, let us denote $(P.16)$ as the condition:

$$U + V(I_t, b_{t-1}) + W < 0 \tag{7}$$

where

$$U = (1+\delta)E + \delta(w_H - w_L)[q_L(p_1 - p_3) + (1-q_L)(p_2 - p_4)] \tag{8}$$

$$V(I_t, b_{t-1}) = \delta(w_H - w_L)(\widetilde{p} - \widetilde{p}_e) \tag{9}$$

$$W = -\{\Pr(e_{t+1} = E/e_t = e_L) - \Pr(e_{t+1} = E/e_t = e_H)\}\delta E \tag{10}$$

Notice that U is completely determined by parameters and W depends on probabilities, i.e. elements of the transition matrix. V depends on income levels I_t (which could be L, M or H) and inherited durables (which could be $0, \beta_{m0}, \beta_{me}, \beta_{h0}$ or β_{he}). Thus, since there may be 3 income levels and 5 durable levels in our model, equation $(P.16)$ (or (8)) encapsulates 15 ($= 3\times5$) conditions. Of these, we know that when the income level is L, households cannot afford education (by assumption) – hence, (8) encapsulates 10 conditions in total, viz conditions valid for 2 income levels $(M, H)$and 5 possible levels of inherited durables.

Now, as in Appendix 2, define $\eta(I_t, b_{t-1}) = 1$ if a household with $t-$period income I_t and inherited durables b_{t-1} finds it optimal to choose $e_t = e_L$, and $\eta(I_t, b_{t-1})$ =0 otherwise (as per equation $(P.16)$).

Then define $t_1 - t_{10}$ as in equations (5) – (24) of Appendix 1. As explained above, each t_i $(i = 1, 2, ..., 10)$ corresponds to the 10 conditions encapsulated in $(P.16)$ (or, (7)).

Now let $tvector = \{t_1, t_2, t_3, ..., t_9, t_{10}\}$ corresponding to any transition matrix and a set of parameters $\{w_H, w_L, p_1, p_2, p_3, p_4, \beta, \sigma, q_L, \delta, E\}$ (assume $C = 2w_L$). We use the following algorithm to specify the transition matrix of the stochastic process (7).

1. Iteration 0 : Assume $W^{(0)} = 0$ (see (8), (11)). Use $W^{(0)}$ to compute $tvector^{(0)}$and define the (25×25) transition matrix $P^{(0)}$, as outlined in Appendix 2. Denote the state corresponding to each row of $P^{(0)}$as $(b_t, b_{t-1}, tvector^{(0)})$. Denote the state corresponding to each column of $P^{(0)}$as $(b_{t+1}, b_t, tvector^{(0)})$.

2. Iteration 1: Use the elements of $P^{(0)}$ to compute $W^{(1)}$; use $W^{(1)}$ to compute $tvector^{(1)}$ and define the (25×25) transition matrix $P^{(1)}$. Denote the state corresponding to each row of $P^{(1)}$as $(b_t, b_{t-1}, tvector^{(0)})$. Denote the state corresponding to each column of $P^{(1)}$as $(b_{t+1}, b_t, tvector^{(1)})$.

3. Iteration 2: Use the elements of $P^{(1)}$ to compute $W^{(2)}$; use $W^{(2)}$ to compute $tvector^{(2)}$ and define the (25×25) transition matrix $P^{(2)}$. Denote the state corresponding to each row of $P^{(2)}$ as $(b_t, b_{t-1}, tvector^{(1)})$. Denote the state corresponding to each column of $P^{(2)}$as $(b_{t+1}, b_t, tvector^{(2)})$.

4. Repeat the iterations till: (a) $tvector$ obtained in two subsequent iterations is the same, or (b) $tvector$ is the same as any $tvector$ previously obtained.

5. Suppose that, at the end of the iterations, we obtain $tnum$ unique values of $tvector$. Stack the matrices $P^{(0)}, P^{(1)}, ..., P^{(tnum-1)}$ appropriately (matching the states indicated in rows and columns of each $P^{(i)}$) to define the composite transition matrix P, which must be of order $(25tnum \times 25tnum)$.
6. Define matrix $Q = P - I(25tnum)$, where $I(25tnum)$ is the unit matrix of order $25tnum$.
7. Define matrix $\widehat{P}$ by replacing the last row of

   $Qwith1's$. (This ensures, in the next step, that the solution vector $x*$ has elements that add to 1.)
8. Solve for $x*$ such that $x * \widehat{P} = x$ (we use 'lusolve' in STATA (MATA) for this step). Note that $x*$ is a vector of order $25tnum$. Each element of $x*$ corresponds to a state $(b_{t-1}, b_t, tvector)$ and denotes the proportion of households who belong to this state in steady state.
9. Map $(b_{t-1} + b_t)$ to the corresponding element of $x*$ to derive the steady state distribution of total durables in households.

Using the procedure above, the steady state distribution $x*$ for Example 1 can be solved to be (graphed in Figure 2):

Total household Durables	Steady State Proportion x*
0	0.342
78.5	0.000
90	0.308
157	0.000
168.5	0.000
180	0.236
247	0.000
258.5	0.009
270	0.083
337	0.013
348.5	0.009
360	0.000